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## Temperature Dependence of Electron Mobility in Si Inversion Layers

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Temperature dependence of electron mobility in Si inversion layers is investigated using channel conductance measurements in the temperature range from 15 to 300 K. The electron mobility is analyzed by using the theoretical model in which two-dimensional electron gas confined in the inversion layer interacts with acoustic phonons, intervalley phonons, surface roughness, and ionized impurities. At higher temperatures above 100 K, the electron mobility is dominated by acoustic phonon scattering and intervalley phonon scattering. At temperatures below 100 K, ionized impurity scattering plays an important role in determining the mobility in the case of low electron sheet densities, whereas in the case of higher electron sheet densities, surface roughness scattering becomes important.

**KEYWORDS:** Si, inversion layer, electron mobility, electron-phonon interaction, two-dimensional electron gas

### §1. Introduction

Electron mobility in Si inversion layers is one of the most important parameters for determining device characteristics of metal-oxide-semiconductor field-effect transistors (MOSFETs), and is also an interesting subject from the viewpoint of two-dimensional structures. Many authors have investigated the mobility experimentally and theoretically.<sup>1)</sup> From these investigations, the observed anisotropy of electron mobility is interpreted satisfactorily in terms of the effective mass anisotropy.<sup>2)</sup> At low temperatures, two mechanisms are known to become important: scattering of electrons in Si inversion layers by Coulomb potential and surface roughness. Stern and Howard<sup>3)</sup> theoretically explained the increase in mobility with increasing surface electron density at low temperatures by the scattering of electrons due to charged centers in the extreme quantum limit (EQL), where they took into account screening of the centers by carriers in the inversion layer. The calculated results have been found to be in qualitative agreement with the experiment of Fang and Fowler.<sup>4)</sup> Hartstein *et al.*<sup>5)</sup> experimentally subtracted the contribution of ionized impurity scattering from their results and succeeded in explaining the surface electron density dependence of the electron mobility in terms of surface roughness scattering at high electron densities. Ando<sup>6)</sup> has made theoretical calculations of the mobility by surface irregularities and showed that the calculated mobility with surface roughness parameters of 4.3 Å high and 15 Å long gives good agreement with the experiments of Hartstein *et al.*<sup>5)</sup> At high temperatures, on the other hand, the scattering due to acoustic phonons and intervalley phonons play an important role. Kawaji<sup>7)</sup> first proposed a model for acoustic phonon scattering based on the interaction between two-dimensional electron gas (2DEG) and phonons, where the relaxation time of electrons was found to be proportional to the average width of the quantized state and the

carrier concentration dependence of the electron mobility observed by Fang and Fowler,<sup>4)</sup>  $\mu \propto N_s^{-1/3}$  was explained well. Ezawa *et al.*<sup>8,9)</sup> proposed the expression for the electron mobility by acoustic phonon scattering, based on the deformation-potential-type interaction between electrons and bulk phonons or surfons. They found that the calculated mobilities vary as  $\mu \propto N_s^{-1/3}$  and  $\mu \propto T^{-1}$ , but the calculated values are larger than the experimental ones by a factor of about 6. Shirahata and Hamaguchi<sup>10)</sup> have calculated the mobility due to acoustic phonon scattering and intervalley phonon scattering, taking into account quantization of electrons. Their analysis is based on the electron-phonon interaction model proposed by Price.<sup>11)</sup> They have shown that the normal field dependence of the electron mobility observed at room temperature by Cooper and Nelson<sup>12)</sup> can be explained well in terms of the quantization. Takagi *et al.*<sup>13)</sup> recently pointed out that at higher normal fields, the electron mobility is limited by surface roughness scattering and at lower surface carrier density, it is limited by Coulomb scattering.

Fang and Fowler<sup>4)</sup> reviewed the observed transport properties of electrons in Si inversion layers, where they showed the temperature-dependence data of the mobility in the range from 4.2 to 300 K for various gate voltages. Kotera *et al.*<sup>14)</sup> have observed the temperature dependence similar to Fang and Fowler and ascribed the variation as  $\mu_{\text{eff}} \propto T^{-1 \sim -1.5}$  at high temperatures above 100 K to surface phonon scattering. Sah *et al.*<sup>15)</sup> have applied a simple two-dimensional electron gas model to the scatterings due to surface oxide charges, acoustic phonons and intervalley phonons. They have compared the calculated results with the observed mobility on a weakly inverted surface at temperatures from 30 to 300 K and found that the mobility limited by phonon scattering varies as  $T^{-1}$  at temperatures below 100 K and  $T^{-2}$  above 150 K. Hartstein *et al.*<sup>16)</sup> have measured the electron mobility as functions of inversion layer carrier density, in-

terfacial  $\text{Na}^+$  ion density, substrate bias voltage and temperature ( $4.2 \leq T \leq 80$  K), where they have succeeded in separating the contributions to the mobility arising from ionized impurity scattering, surface roughness scattering and phonon scattering by using Matthiessen's rule. These results suggest that the electron mobility in the Si inversion layer is interpreted in terms of the combined effect on the inversion layer carriers interacting with the various types of scatterers, such as acoustic phonon scattering, intervalley phonon scattering, surface roughness scattering and ionized impurity scattering.

In our previous paper,<sup>17)</sup> we proposed an analytical expression for the electron mobility in the Si inversion layer, where the quantization of electrons and four scattering mechanisms (acoustic phonon, intervalley phonon, surface roughness and ionized impurity) are taken into account. The effective normal field dependence of the mobility at 300 and 77 K is found to be well explained by the model. It is therefore very interesting to investigate temperature dependence of the electron mobility. In this paper, we report the temperature dependence of the electron mobility in the Si inversion layer measured by DC channel conductance methods at temperatures from 15 to 300 K. The experimental data are analyzed by means of the model used for the analysis of the effective normal field dependence of the electron mobility at 300 and 77 K. In §2, we present experimental procedures and results. In §3, the observed mobility is compared with the theoretical calculations, and we summarize the present results.

## §2. Experimental and Results

The  $n$ -channel MOSFETs with channel length  $L$  of 200  $\mu\text{m}$  and width  $W$  of 100  $\mu\text{m}$  were fabricated on (100) Si wafers with the substrate doping of  $N_A = 8 \times 10^{16} \text{ cm}^{-3}$ . The channel current flows along the  $\langle 110 \rangle$  direction. The specimens had three pairs of voltage probes on both sides of the channel with an equal spacing of 50  $\mu\text{m}$ . The gate oxide thickness  $T_{\text{ox}}$  was 25 nm. As pointed out in our previous paper,<sup>17)</sup> measurements of the electron mobility by Hall effect and DC conductance methods revealed that the surface electron densities and the electron mobilities obtained by the two different methods were found to be in good agreement with each other, except that the effective mobility is slightly larger than the Hall mobility in the range of low surface electron density at 77 K. In the present experiment, therefore, the mobility was obtained by the DC conductance method.<sup>18,19)</sup> We kept the source-drain voltage  $V_d$  at a constant value of 50 mV to achieve the linear region in the temperature range from 15 to 300 K.

Effective mobility  $\mu_{\text{eff}}$  can be estimated from channel conductance and is given by

$$\mu_{\text{eff}} = (LI_d) / (WV_d e N_s), \quad (2.1)$$

where  $I_d$  is the drain current,  $e$  is the magnitude of electron charge and the inversion layer sheet electron density  $N_s$  is given by

$$N_s = C_{\text{ox}}(V_g - V_{\text{th}}) / e. \quad (2.2)$$

Here,  $C_{\text{ox}}$  is the gate oxide capacitance per unit area,  $V_g$

the gate voltage and  $V_{\text{th}}$  the threshold voltage. We analyze the effective mobility in the region of  $N_s > 5 \times 10^{11} \text{ cm}^{-2}$ , because it is difficult to estimate an accurate surface carrier density in the region near the threshold voltage.

Figure 1 shows effective mobilities as a function of surface electron densities at different temperatures from 15 to 300 K. As seen in Fig. 1, the effective mobility increases as the temperature decreases. At high temperatures, the electron mobility decreases monotonously with

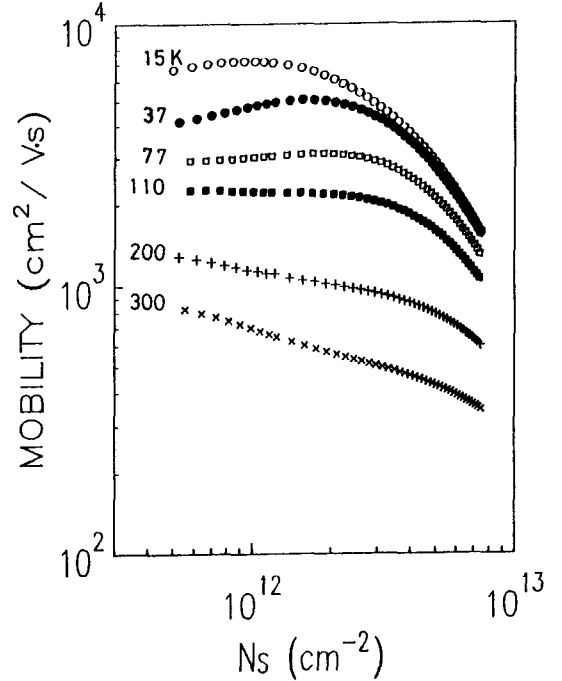


Fig. 1. Measured electron mobility as a function of sheet electron density at different temperatures from 15 K to 300 K.

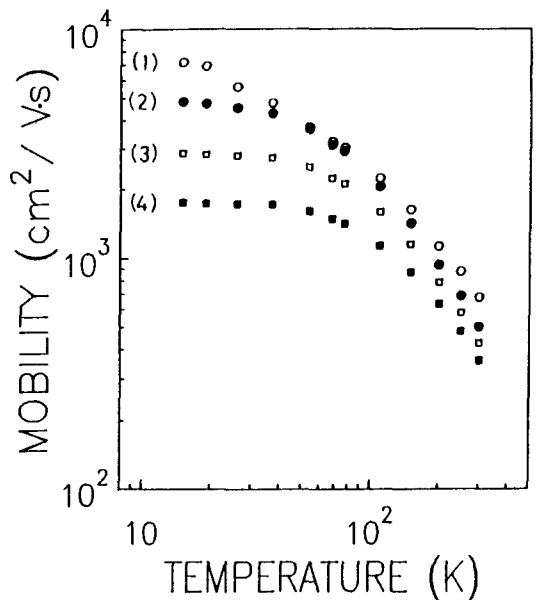


Fig. 2. Effective mobility as a function of temperature for surface electron densities of (1)  $N_s = 1 \times 10^{12} \text{ cm}^{-2}$ , (2)  $3 \times 10^{12} \text{ cm}^{-2}$ , (3)  $5 \times 10^{12} \text{ cm}^{-2}$ , (4)  $7 \times 10^{12} \text{ cm}^{-2}$ .

increasing surface electron density, while at low temperatures, the mobility first increases and then rapidly decreases at higher electron densities. The data in Fig. 1 are represented in Fig. 2 as a function of temperature for several surface electron densities  $N_s = 1 \times 10^{12} \text{ cm}^{-2}$ ,  $3 \times 10^{12} \text{ cm}^{-2}$ ,  $5 \times 10^{12} \text{ cm}^{-2}$ ,  $7 \times 10^{12} \text{ cm}^{-2}$ . At high temperatures above 100 K, the measured mobility approaches an asymptote expressed as  $T^{-n}$  ( $n > 1$ ). At low temperatures below 100 K, on the other hand, the temperature dependence of electron mobility depends strongly on the surface electron densities.

It is well known that the electron mobility is dominated by acoustic phonon and intervalley phonon scattering at high temperatures, whereas Coulomb scattering and surface roughness scattering play an important role at low temperatures. In the following section, we will analyze the experimental results by using the theoretical model, which takes into account these four principal scattering mechanisms and the electron quantization effect.

### §3. Analysis and Discussion

It has been shown by Sabnis and Clemens,<sup>20)</sup> Sun and Plummer<sup>19)</sup> and very recently by Takagi *et al.*<sup>13)</sup> that the effective mobility can be expressed by a universal curve when it is plotted as a function of the effective normal field defined by

$$E_{\text{eff}} = e(0.5N_s + N_{\text{depl}}) / \kappa_{\text{Si}} \epsilon_0, \quad (3.1)$$

where  $N_{\text{depl}}$  is the charge density per unit area in the depletion layer,  $\kappa_{\text{Si}}$  the dielectric constant of Si and  $\epsilon_0$  the free space permittivity. Taking into account their results, we plot the effective mobility as a function of effective normal field at temperatures from 15 to 300 K in Fig. 3. The broken line in Fig. 3 indicates an  $E_{\text{eff}}^{-2}$  dependence which is caused by surface-roughness scattering (see appendix). We find in Fig. 3 that, at high temperatures, the effective mobility monotonously decreases as the effective field increases, whereas at low temperatures, the mobility first rises and then rapidly decreases as  $E_{\text{eff}}^{-2}$ . We will show a comparison of the experimental results with calculations.

In the previous paper,<sup>17)</sup> we proposed analytical expressions for the electron mobility in the Si inversion layer. The analytical model is based on the interaction of 2DEG confined in the inversion layer with acoustic phonons, intervalley phonons,<sup>11)</sup> surface roughness,<sup>11)</sup> and ionized impurities.<sup>21)</sup> The model includes the anisotropy of effective mass, many valley structures and three kinds of *g*- and *f*-type intervalley phonons.<sup>22)</sup> The electronic states of the 2DEG are calculated by solving the Schrödinger equation and the Poisson equation self-consistently using the method of Stern.<sup>23)</sup> Numerical calculations are carried out to obtain eigen-values, envelope functions and subband sheet electron densities. From these numerical calculations, we found that at 300 K, more than 85% of the total electrons in the Si inversion layer occupy the lower two electric subbands,  $E_0$  and  $E_{0'}$ , which arise from two valleys with the longitudinal effective mass  $m_0(0.19m_0)$  and from four valleys with the transverse effective mass  $m_t(0.92m_0)$ , respectively, on the (100) Si surface. From this reason, we take into account the elec-

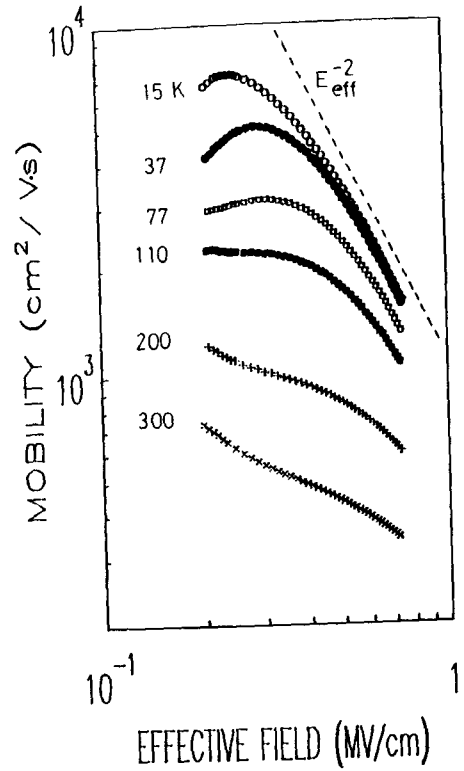


Fig. 3. Effective channel mobility is plotted as a function of effective normal field at different temperatures from 15 K to 300 K.

trons in these two subbands for calculating the mobility. At lower temperatures, the population of electrons at the lowest subband increases, and thus the above assumption becomes more reasonable. The relaxation time for each scattering may be obtained by using these numerically calculated results, as shown in our previous paper.<sup>17)</sup> Since the density of states is independent of energy for 2DEG, the energy-averaged relaxation time  $\langle \tau_p \rangle$  is easily calculated for the combined relaxation time of acoustic phonon, intervalley phonon and surface roughness scattering. At low temperatures, impurity scattering plays an important role in determining mobility. The relaxation time for the impurity scattering<sup>21)</sup> is very complicated, and the evaluation of the average value  $\langle \tau_{\text{ion}} \rangle$  was made numerically. We combine the two average relaxation times with an approximation method using the formula

$$\frac{1}{\langle \tau_0 \rangle} = \frac{1}{\langle \tau_{\text{ion}} \rangle} + \frac{1}{\langle \tau_{\text{ps}} \rangle}, \quad (3.2)$$

where  $\langle \tau_0 \rangle$  is the average relaxation time for electrons in the  $E_0$  subband. The average relaxation time for electrons in the  $E_{0'}$  subband is also calculated in the same manner. We assume that the average mobility  $\mu$  is determined by the electrons in subbands  $E_0$  and  $E_{0'}$  using the expression

$$\mu = \frac{N_0 e \langle \tau_0 \rangle / m_c + N_{0'} \langle \tau_{0'} \rangle / m_{c'}}{N_0 + N_{0'}}, \quad (3.3)$$

where  $m_c (= m_0)$  and  $m_{c'} (= 2m_t m_0 / (m_0 + m_t))$  are the conductivity masses of subbands  $E_0$  and  $E_{0'}$ .  $N_0$  and  $N_{0'}$  are electron sheet densities in subbands  $E_0$  and  $E_{0'}$ , respectively. With these assumptions, the evaluation of the electron mobility is straight-forward. In the previous paper,

we found a reasonable agreement between the calculated results and the experimental data at 300 and 77 K.

Figure 4 shows the mobility as a function of effective normal field at 15 K, where we compare the measured effective mobility (solid circles) with the calculated mobility (solid curve). A value of  $D_{ac} = 12$  eV is taken for the deformation potential of acoustic phonon scattering.<sup>9)</sup> For the physical interpretation, we plotted the calculated mobility due to the combined scattering of acoustic phonon, intervalley phonon and surface roughness scattering by the dot-dashed curve and the mobility due to ionized impurity scattering by the double dot-dashed curve. These theoretical curves in Fig. 4 are calculated using the impurity concentration of  $N_A = 8 \times 10^{16} \text{ cm}^{-3}$  and surface roughness parameter of  $\Delta A = 25 \times 10^{-20} \text{ m}^2$ . We found that the calculated mobility is in good agreement with experiments. At low effective fields, the decrease in mobility is interpreted in terms of the insufficient screening of ionized impurities by the inversion electrons. On the other hand, at higher effective fields, the mobility decreases as  $E_{eff}^{-2}$  which is well explained in terms of surface roughness scattering (see appendix).

Figures 5(a) and 5(b) show the temperature dependencies of the electron mobility for the sheet electron densities

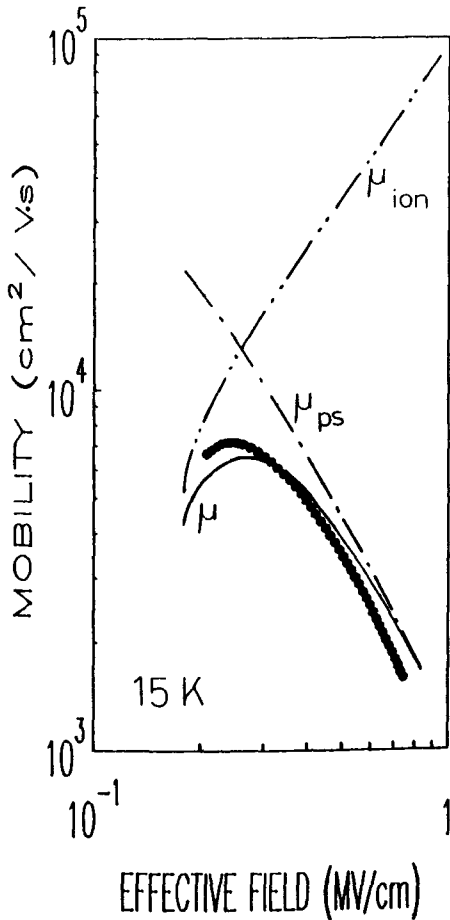
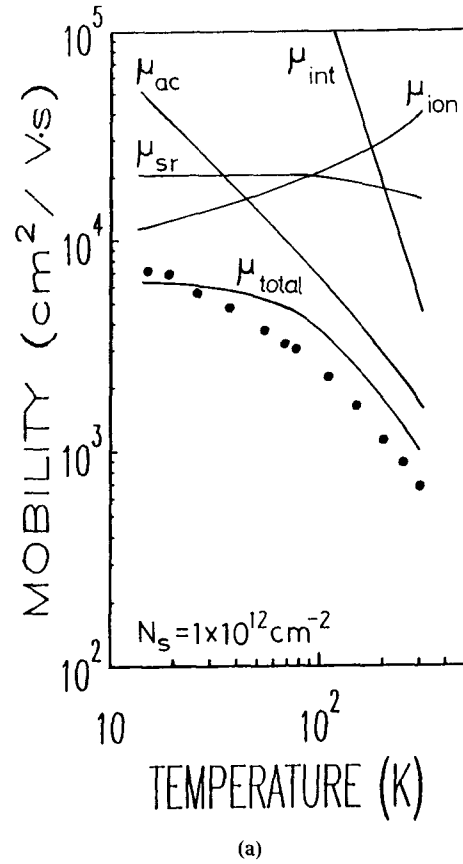
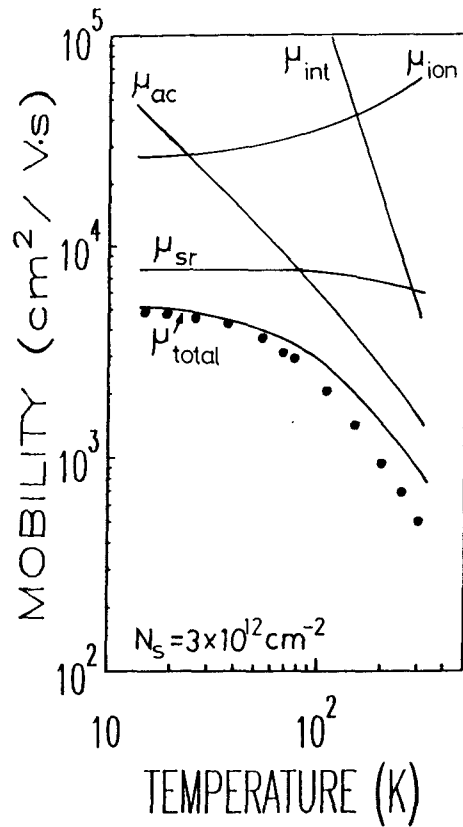


Fig. 4. Effective normal field dependence of the measured effective mobility (solid circles) is compared with the calculated mobility at 15 K. The dot-dashed curve ( $\mu_{ps}$ ) is calculated mobility due to acoustic phonons, intervalley phonons and surface roughness scattering, while the double dot-dashed curve ( $\mu_{ion}$ ) is the calculated mobility due to the ionized impurity scattering alone.



(a)



(b)

Fig. 5. Results of numerical calculations of temperature dependence of electron mobility (solid lines) for  $N_s = 1 \times 10^{12} \text{ cm}^{-2}$  and  $3 \times 10^{12} \text{ cm}^{-2}$ , respectively, together with the experimental data obtained by conductance measurements (solid circles). (a)  $N_s = 1 \times 10^{12} \text{ cm}^{-2}$  (b)  $N_s = 3 \times 10^{12} \text{ cm}^{-2}$

sities of  $N_s = 1 \times 10^{12} \text{ cm}^{-2}$  and  $3 \times 10^{12} \text{ cm}^{-2}$ , respectively. The solid circles in Figs. 5(a) and 5(b) are the experimental data obtained by the conductance method and the solid line ( $\mu_{\text{calc}}$ ) is the calculated result with impurity concentration  $N_A = 8 \times 10^{19} \text{ cm}^{-3}$  and surface roughness parameter  $\Delta = 25 \times 10^{-20} \text{ m}^2$ . For the purpose of physical interpretation, we also plotted the mobility due to separate scattering processes, which are acoustic phonon scattering ( $\mu_a$ ), intervalley phonon scattering ( $\mu_m$ ), surface roughness scattering ( $\mu_r$ ) and ionized impurity scattering ( $\mu_{\text{imp}}$ ). It is clear from these results that at low temperatures below 100 K, the electron mobility is limited by the ionized impurity scattering at low electron densities (see Fig. 5(a) for  $N_s = 1 \times 10^{12} \text{ cm}^{-2}$ ), and the surface roughness scattering plays an important role instead of the ionized impurity scattering with increasing surface carrier density (see Fig. 5(b) for  $N_s = 3 \times 10^{12} \text{ cm}^{-2}$ ). At lower electron densities, the ionized impurities are not sufficiently screened and surface roughness scattering is weaker because of the lower effective electric field, resulting in the mobility being limited by the ionized impurity scattering. Such a feature is obvious in Figs. 5(a) and 5(b). At higher temperatures above 100 K, the measured mobility varies as  $T^{-n}$  ( $n > 1$ ), despite the surface electron density, which shows good agreement with the data of Fang and Fowler<sup>4)</sup> and the experimental result by Kotera *et al.*<sup>11)</sup> ( $\mu \propto T^{-1} - T^{-1.5}$ ). This is evident from the calculated results shown in Figs. 5(a) and 5(b), where we find that the temperature dependence of the electron mobility near 300 K is determined by intervalley phonon scattering in addition to acoustic phonon scattering. Our present calculations show that the mobility ( $\mu_a$ ) due to acoustic phonon scattering alone is proportional to  $T^{-1}$  and that the temperature dependence of the mobility ( $\mu_m$ ) due to intervalley phonon scattering alone is approximated by the relation  $T^{-1}$ . It is found from the present work that the temperature dependence of the electron mobility in the Si inversion layer is well explained by the previous model,<sup>12)</sup> when we include four principal scattering mechanisms, acoustic phonons, intervalley phonons, surface roughness and ionized impurities.

The present results are summarized as follows. Temperature dependence and effective normal field dependence of the electron mobility were measured by means of the channel conductance method in the temperature range from 15 to 300 K for surface electron densities between  $5 \times 10^{11}$  and  $8 \times 10^{12} \text{ cm}^{-2}$ . These experimental data were analyzed through the model based on the interaction between the 2DEG confined in the inversion layer and acoustic phonons, intervalley phonons, surface roughness and ionized impurities. The calculated mobilities are found to be in good agreement with the observations. At temperatures above 100 K, the temperature dependence of the electron mobility is  $T^{-n}$  ( $n > 1$ ), interpreted in terms of the phonon scattering. At temperatures below 100 K, on the other hand, the electron mobility for low electron densities is limited by the ionized impurity scattering and the screening effect was found to be very important. The surface roughness scattering becomes important with increasing surface electron density or effective normal electric field.

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## Appendix

Surface irregularities at the semiconductor-insulator interface are expected to constitute a major cause of scattering, especially at high normal fields. This surface roughness scattering has been theoretically investigated within a simple model.<sup>6)</sup> The model assumes that only the lowest electric subband is occupied by electrons, and deviations of the interface from flatness  $\Delta(x, y)$  are small and slowly varying. The effect of the surface roughness is transferred from an interface potential barrier  $V(z)$  to other parts of Hamiltonian  $H'$ , which can be dealt with as perturbations. Hamiltonian  $H'$  due to the surface roughness is approximately given to the lowest order in  $\Delta(x, y)$  by

$$H' = -\frac{dV}{dz} \Delta(x, y), \quad (\text{A} \cdot 1)$$

where the  $z$ -axis is taken perpendicularly to the regular surface. The procedure of calculations of the relaxation time due to surface roughness scattering is as follows.

For a matrix element of the Hamiltonian  $H'$ , we have

$$\begin{aligned} \langle \mathbf{k}' | H' | \mathbf{k} \rangle &= -\int_{-\infty}^{\infty} \zeta^*(z) \frac{dV}{dz} \zeta(z) dz \\ &\quad \times \frac{1}{A} \int_A \Delta(x, y) e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{R}} d^2R \\ &= -F_{\text{eff}} \Delta(\mathbf{k}-\mathbf{k}'), \end{aligned} \quad (\text{A} \cdot 2)$$

where

$$F_{\text{eff}} = \int_{-\infty}^{\infty} \zeta^*(z) \frac{dV}{dz} \zeta(z) dz, \quad (\text{A} \cdot 3)$$

$$\Delta(\mathbf{q}) = \frac{1}{A} \int_A \Delta(\mathbf{q}) e^{i\mathbf{q} \cdot \mathbf{R}} d^2R, \quad (\text{A} \cdot 4)$$

$|\mathbf{k}|$  is the initial state of an electron wave vector  $\mathbf{k}'$ ,  $|\mathbf{k}'\rangle$  the final state of an electron wave vector  $\mathbf{k}'$ , the envelope function  $\zeta(z)$  obtained by self-consistent calculations of the coupled Schrödinger and Poisson equations,  $A$  the area of normalization for the wave functions,  $\mathbf{R}$  the point vector parallel to the interface and  $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ .  $F_{\text{eff}}$  has been derived by Matsumoto and Uemura<sup>24)</sup> as

$$\begin{aligned} F_{\text{eff}} &= \frac{e^2}{\kappa_{\text{Si}} \epsilon_0} \left( \frac{1}{2} N_s + N_{\text{depl}} \right) \\ &= e E_{\text{eff}}. \end{aligned} \quad (\text{A} \cdot 5)$$

$\Delta(\mathbf{R})$  is assumed to have a Gaussian form of the correlation between two points:

$$\langle \Delta(\mathbf{R}) \Delta(\mathbf{R} + \mathbf{R}_1) \rangle = \Delta^2 e^{-R_1/\Lambda^2}, \quad (\text{A} \cdot 6)$$

where  $\langle \dots \rangle$  means the sample average,  $\Delta$  is the root mean square deviation of the interface and  $\Lambda$  the correlation length. In this way,  $|\Delta(\mathbf{q})|^2$  is obtained as

$$|\Delta(\mathbf{q})|^2 = \frac{\pi \Delta^2 \Lambda^2}{A} e^{-q^2 \Lambda^2 / 4}. \quad (\text{A} \cdot 7)$$

Under these assumptions and neglecting the screening effect, the relaxation time  $\tau_{sr}$  of the electron with the wave vector  $\mathbf{k}$  is given by

$$\frac{1}{\tau_{sr}(\mathbf{k})} = \frac{2\pi}{\hbar} \sum_{\mathbf{k}'} |\langle \mathbf{k}' | H' | \mathbf{k} \rangle|^2 \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}) (1 - \cos \theta), \quad (\text{A} \cdot 8)$$

where  $\varepsilon_{\mathbf{k}}(\varepsilon_{\mathbf{k}'})$  is the energy of the electron with the wave vector  $\mathbf{k}(\mathbf{k}')$ ,  $\theta$  the angle between  $\mathbf{k}$  and  $\mathbf{k}'$  and  $\delta(x) = 1$  if  $x=0$ ,  $\delta(x)=0$  otherwise. We replace the quadratic part of the matrix element of the Hamiltonian  $H'$  in eq. (A·8) with eqs. (A·2)–(A·7) and obtain the following relation:

$$\frac{1}{\tau_{sr}} = \frac{2\pi}{\hbar} \sum_q [\Delta \Lambda e E_{\text{eff}}]^2 [\exp(-q^2 \Lambda^2 / 4)] \times \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}-q}) (1 - \cos \theta). \quad (\text{A} \cdot 9)$$

As shown by Hartstein *et al.*,<sup>5)</sup> the correlation length  $\Lambda$  is about 6 Å and the quantity of  $k\Lambda$  is much less than unity. In this case, the relaxation time for surface roughness scattering is approximated by

$$\frac{1}{\tau_{sr}} = \pi m_d [\Delta \Lambda e E_{\text{eff}}]^2 / \hbar^3, \quad (\text{A} \cdot 10)$$

where  $m_d$  is the effective density of states mass. Equation (A·10) shows that the electron mobility limited by surface roughness scattering varies as  $E_{\text{eff}}^{-2}$ .

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